What is a Query Language?

Universality of Data Retrieval Languages, Aho and Ullman, POPL 1979

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What is …?

- What Is A Query Language?
  - A language that allows retrieval and manipulation of data From a database.
- What Is A Database?
  - A large collection of DATA
  - The data can be grouped into sets whose elements have similar structure.
- What Kind of Structure Can the Data Have?
- What Kind of Manipulation Should Be Allowed?

Some Ideas

- Relations should be treated as sets of tuples.
- The query language must have a simple, non-operational meaning that is independent of physical data representation.
- There must be efficient ways to process queries over (large) sets of similarly structured facts.

We will focus on the relational model

*Principles for A Relational Query Language**

* Proposed by Aho & Ullman

1) Relation = Set of Tuples.
   Ordering & other storage details should not be visible.
2) Data Values should not be 'Interpreted'.

$$\text{Def : Let } \mu = D \rightarrow D \text{ be a Bijection.}$$

A Function f is Allowable if :

$$\mu(f(r_1,\ldots,r_n)) = f(\mu(r_1),\ldots,\mu(r_n))$$

Note: (2) Says that no special meaning should be attached to data values (as far as the query language is concerned); thus, Arithmetic is Disallowed!

5+6 = 11, 8<9, …

Principles – Refinement

- Principle (2) is too restrictive.
- Relax it slightly:

Let P be a special set of predicates. (e.g. <, =)

μ <u>Preserves</u> P if $\forall p \in P$

$\mu(p(x_1,\ldots, x_n))$ is true $\Leftrightarrow$ $p(\mu(x_1),\ldots, \mu(x_n)$ is true.

Relaxing Principle (2) : We require that :

$\mu(f(r_1,\ldots, r_n)) = f(\mu(r_1),\ldots, \mu(r_n))$

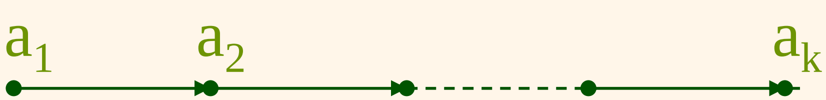
only for Bijections μ that preserve P.

Note: If we include +, $\times$, etc. to *P*, soon only the identity function will preserve *P*!

Allowable Fns – Transitive Closure

- Aho & Ullman’s notation of allowable function is rather restrictive. However:
  1. All Relational Algebra queries are allowable.
  2. Transitive Closure is allowable.
- And they prove that:
  - *There is no Relational Algebra query that computes the Transitive Closure of a Relation.*

Any R.A expression has a fixed size, say n. Choose Relation R:



$k > n$

The relational algebra expression cannot deal with (a_1, a_k).

Proposal

- We should extent RA to support a *least fixpoint* operator.
  - Leads to recursive queries
  - Some systems (e.g., Oracle) support limited forms of recursion like transitive closure. Others (DB2) support linear recursion, following SQL:1999.

Least Fixpoints

- The LFP operator is defined as follows:

$$LFP(R = f(R)) = r, \text{ where:}$$
$$1.\ r = f(r)$$
$$2.\ \text{if } r' = f(r') \text{ then } r \subseteq r'$$

- Theorem (Tarski):
  There is a least fixpoint satisfying *LFP(R=f(R))* if '*f*' is *monotone*.

  $$\text{Monotone}: r_1 \subseteq r_2 \Rightarrow f(r_1) \subseteq f(r_2)$$

  Note: If 'f' is a relation algebra expression without '−' (set diff.), then it is monotone.

Least Fixpoint – Cont.

- Theorem (Kleene)

  If f is continuous & over a complete lattice,

  $$LFP(R = f(R)) = \lim_{n\to\infty} f^n(\emptyset)$$

- Example: Transitive Closure

$$R = R \circ r \bigcup r;$$

$$\therefore\ f(R) \text{ is } R \circ r \bigcup r$$

$$f(\emptyset) = r;$$

$$f(f(\emptyset)) = f(r) = r \circ r \bigcup r$$

$$\vdots$$

$$f^n(\emptyset) = \bigcup_{i=1}^{n} r \circ r \circ \cdots \circ r$$

LFP - Cont.

- Claim:
  The LFP operator satisfies principles 1&2

- Theorem (Aho-Ullman):
  There is no relational algebra expression *E(R)* that computes the transitive closure of an *arbitrary* input relation R.

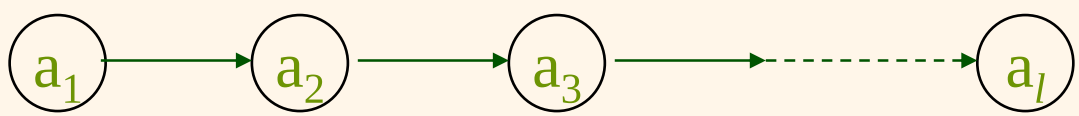
Proof

Consider a set of l arbitrary symbols:

$$\Sigma_l = \{a_1, a_2, \cdots a_l\}$$

We consider a family of relations

$$R_l = \{(a_1, a_2), (a_2, a_3) \cdots (a_{l-1}, a_l)\}$$



We show that NO relational algebra expression computes exactly the tuples in R_l^+ for all l

We will prove that every R.A. expr. $E(R_l)$ can be expressed as : $\{b_1 b_2 \cdots b_k \mid \Psi(b_1, b_2, \cdots b_k)\}$

Where

Ψ is of the form : clause1 $\vee$ clause2 $\vee \cdots$

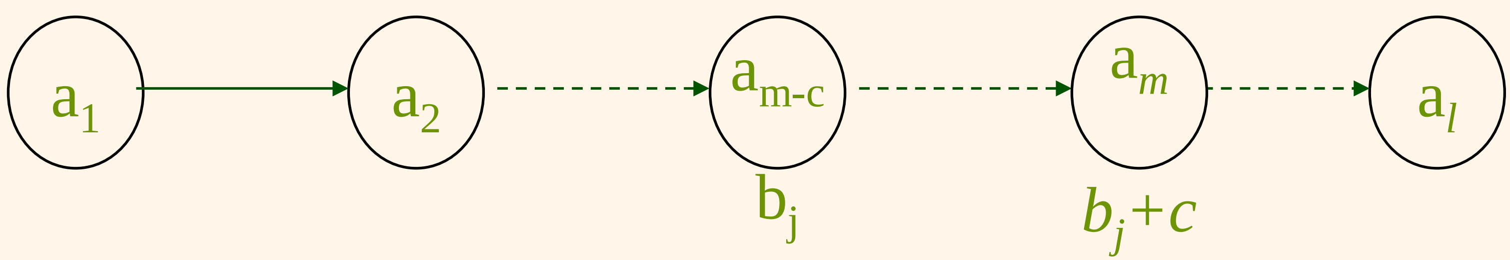
Each clause is of the form : atom1 $\wedge$ atom2 $\wedge \cdots$

Each atom is of the form :

$b_i = a_c, b_i \neq a_c, b_i = b_j + c, b_i \neq b_j + c$

The b's are variables taking values from Σ_l,

and the c's are constants $(0 \leq c \leq l)$



Note: Here $(b_j+c) \equiv a_m$ s.t. $b_j=a_{m-c}$

Lemma : If E is any R.A. expr.

$E(R_l) = \{b_1 b_2 \cdots b_k \mid \Psi(b_1, b_2, \cdots b_k)\}$

Suppose the lemma is true, we can then prove the theorem as follows :

Suppose $E(R) = R^+$, for some E, for all R, then $R_l^+ = \{b_1 b_2 \mid \Psi(b_1, b_2)\}$

<u>Case 1</u> : Every clause in Ψ has an atom of the form :

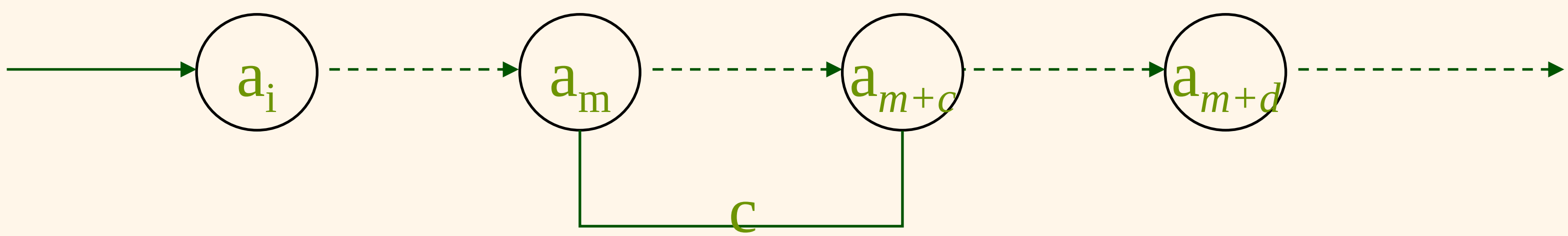
$b_1 = a_i$, $b_2 = a_i$, or $b_1 = b_2 + c$

Consider $(b_1, b_2) = (a_m, a_{m+d})$ where

$m > \forall i$ s.t. $b_1 = a_i$ or $b_2 = a_i$ is an atom;

$d > \forall c$ s.t. $b_1 = b_2 + c$ is an atom

$\therefore (a_m, a_{m+d})$ is not computed, but is in R_l^+



$\underline{\text{Case 2}}$: Some clause in Ψ has ONLY atoms with $\neq$

Consider $(b_1, b_2) = (a_{m+d}, a_m)$

Where no atom

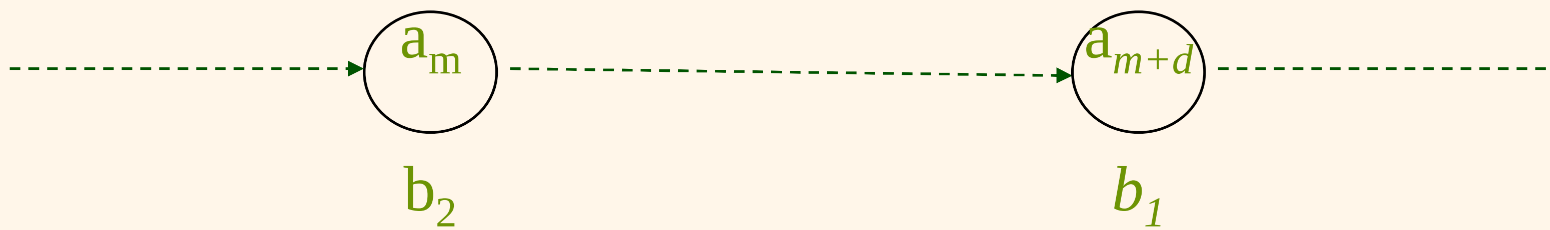
$b_i \neq a_m$ or $b_i \neq a_{m+d}$

appears in Ψ, and

$d > c$, for all c s.t. $b_1 \neq b_2 + c$ or $b_2 \neq b_1 + c$

appears in Ψ.

$\therefore (a_{m+d}, a_m)$ is computed, but is not in R_l^+



Proof of lemma

Basis : 0 operators. $\therefore$ E(R) is R or constant relation.

$R = \{b_1 b_2 | b_2 = b_1 + 1\};$

$\{c_1, c_2, \cdots c_m\} = \{b_1 | b_1 = c_1 \vee b_1 = c_2 \vee \cdots\}$

Induction :

$E = E_1 \cup E_2,\ E_1\text{-}E_2 \ or\ E_1 \times E_2$

$E_1 = \{b_1 \cdots b_k | \Psi_1(b_1 \cdots b_k)\}$

$E_2 = \{b_1^{'} \cdots b_k^{'} | \Psi_2(b_1^{'} \cdots b_k^{'})\}$

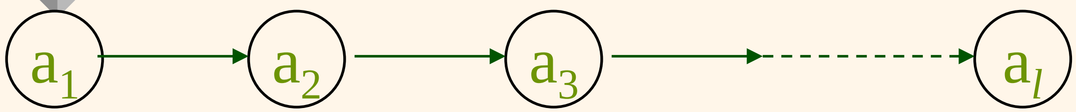
$E_1 \cup E_2 = \{b_1 \cdots b_k | \Psi_1(b_1 \cdots b_k) \vee \Psi_2(b_1 \cdots b_k)\}$

$E = \sigma_F(E_1)$, F has only $=$, $\neq$

$\therefore E = \{b_1 \cdots b_k | \Psi_1(b_1 \cdots b_k) \wedge F(b_1 \cdots b_k)\}$

$E = \pi_S(E_1)$, proceeding similarly …

Transitive closure - more

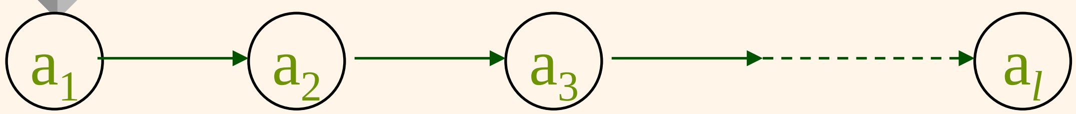


$$R_l = \{(a_1, a_2), (a_2, a_3) \cdots (a_{l-1}, a_l)\}$$

$$\sigma_{1<2}(\pi_1(R_l) \times \pi_2(R_l))$$

Does this relational algebra expr. computes R_l^+ ?

Transitive closure - more

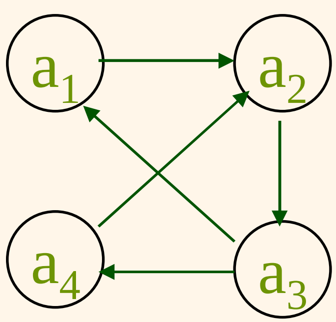


$$R_l = \{(a_1, a_2), (a_2, a_3) \cdots (a_{l-1}, a_l)\}$$

$$\sigma_{1<2}(\pi_1(R_l) \times \pi_2(R_l))$$

Does this relational algebra expr. computes R_l^+ ?

YES! But it is NOT a relation algebra expression!



What does “a_i<a_j” mean now?!

BP-Completeness

- A query language is BP-complete if:
  - All functions that can be expressed in the language are allowable.
  - Let r_1 and r_2 be two relations (instances), such that for all renamings μ

$$r_1 = \mu(r_1) \Rightarrow r_2 = \mu(r_2)$$

Then there is a function f in the language such that

$$r_2 = f(r_1)$$

Example of BP-Complete

A

5	6
6	5
7	8

B

5	6
6	5
10	11

C

5	6
7	8

D

5	6
6	5
7	8
8	7

E

5	6
6	5

F

5	6
6	5
7	8
5	5
6	6

1. If ‘A’ is used as ‘r_1’ in previous slide, which of the others qualifies as ‘r_2’?
2. For each such relation, find relational algebra function *f*.