Evaluation of Recursive Queries

Part 1: Efficient fixpoint evaluation

“Seminaïve Evaluation”

Bottom-up evaluation

- Naïve

  Repeat

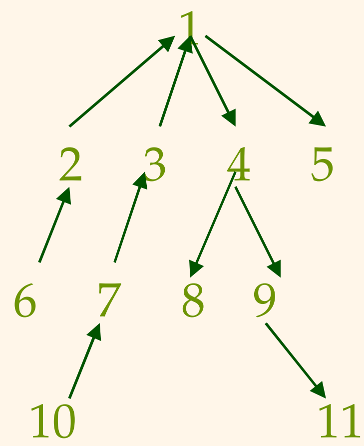
  Apply all rules

  Until no new tuples generated

- Seminaïve
  - If a rule is applied in iteration N, at least one body fact must be a fact generated in iteration N-1 (and not before!).
  - No application is repeated.

Example

up(2,1)　down(1,4)
up(3,1)　down(1,5)
up(6,2)　down(4,8)
up(7,3)　down(4,9)
up(10,7)　down(9,11)



r1: sg(X,Y) :- up(X,Z), down(Z,Y)

r2: sg(X,Y) :- up(X,Z_1), sg(Z_1,Z_2), down(Z_2,Y)

r3: sg(6,Y) ?

Naïve Evaluation proceed as follows:

Step(1)
sg(2,4), sg(2,5), sg(3,4), sg(3,5)

Step(2)
Iteration 1
sg(6,8), sg(6,9), sg(7,8), sg(7,9)

Iteration 2
sg(6,8), sg(6,9), sg(7,8), sg(7,9), sg(10,11)

Iteration 3
No new tuples

Seminaïve Evaluation proceed as follows:

Step(1)
sg(2,4), sg(2,5), sg(3,4), sg(3,5)

Step(2)
Iteration 1
sg(6,8), sg(6,9), sg(7,8), sg(7,9)

Iteration 2
sg(10,11)

Iteration 3
No new tuples

Notation

- Recursive Predicate
  - $p \rightarrow^* p$
- Mutually recursive predicate
  - $p \rightarrow^* q$, $q \rightarrow^* p$
- Strongly connected component (SCC)
  - A maximal set of mutually recursive predicates.
- Linear Rule
  - Only 1 body literal is mutually recursive with head predicate.

```
p(x)      :- q1(x), q2(y).
q1(x)     :- q2(x).
q2(x)     :- q1(y), b(x,y).
q2(x)     :- c(x,y).
c(x,y)    :- d(x), d(y).
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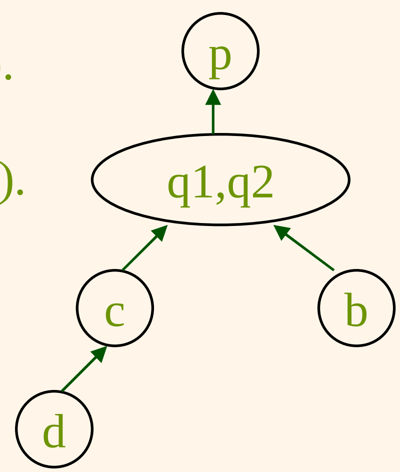


- Program Graph
  - Node = SCC
  - ARC : The 'Depends on' relation '→'

Seminaïve evaluation

- There are two components:
  - Rule Rewriting: Each rule in the program is replaced by a set of rules as follows:

$$p \text{ :- } \underbrace{p_1, p_2, \ldots, p_n}_{\text{recursive w.r.t. p}}, \underbrace{q_1, q_2, \ldots, q_m}_{\text{base predicates}}$$

is replaced by

$\delta p^{new}() \text{ :- } \delta p_1^{old}, p_2, \ldots, p_n, q_1, q_2, \ldots, q_m.$

$\delta p^{new}() \text{ :- } p_1^{old}, \delta p_2^{old}, \ldots, p_n, q_1, q_2, \ldots, q_m.$

…

$\delta p^{new}() \text{ :- } p_1^{old}, p_2^{old}, \ldots, p_{n-1}^{old}, \delta p_n^{old}, q_1, q_2, \ldots, q_m.$

Special case: n=0, i.e. no recursive predicates

$\delta p^{new}() \text{ :- } q_1, q_2, \ldots, q_m.$

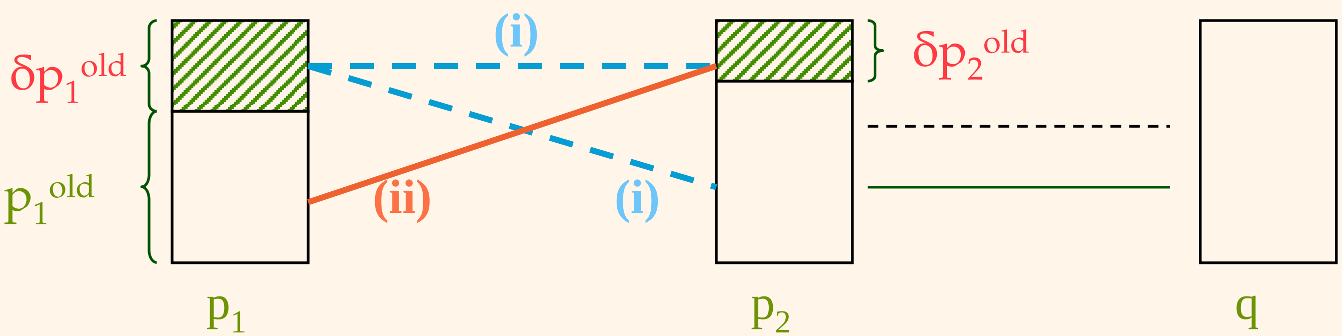
Example

$$p(x,z) \text{ :- } \underbrace{p_1(x,y),\ p_2(y,z)}_{\text{recursive w.r.t. } p},\ \underbrace{q(z,w)}_{\text{base predicates}}$$

is replaced by

(i). $\delta p^{new}(x,z) \text{ :- } \delta p_1^{old}(x,y),\ p_2(y,z),\ q(z,w).$

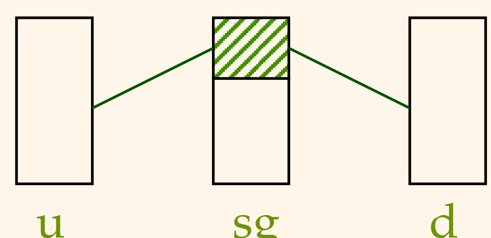
(ii). $\delta p^{new}(x,z) \text{ :- } p_1^{old}(x,y),\ \delta p_2^{old}(y,z),\ q(z,w).$



Special case – 'Linear' rule:

$$sg(x,z) \text{ :- } \underbrace{u(x,v)}_{\text{base}},\ sg(v,w),\ \underbrace{d(w,z)}_{\text{base}}.$$

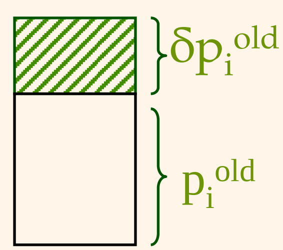
$\delta sg^{new}(x,z) \text{ :- } u(x,v),\ \delta sg^{old}(v,w),$
$d(w,z).$



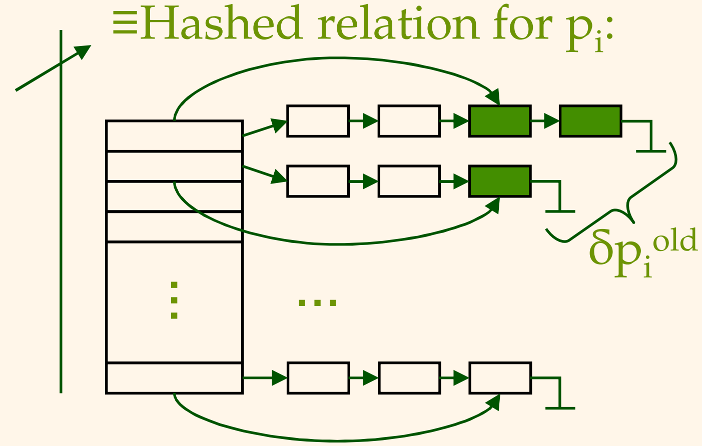
Seminaïve evaluation – Part 2

- Rule Evaluation
  - Repeatedly apply rule in 'iterations' until no new facts.
  - Iteration 1---Use all rules
  - Later iterations---Use only recursive rules
- In each iteration:
  - Apply rules
  - For each non-base predicate p, update associated relations as follows:

$$\left\{\begin{array}{l} p_i^{old} = p_i^{old} \cup \delta p_i^{old} \\ \delta p_i^{old} = \delta p_i^{new} - p_i^{old} \\ p_i = p_i^{old} \cup \delta p_i^{old} \\ \delta p_i^{new} = \emptyset \end{array}\right. \xrightarrow{\text{after}}$$



≡Hashed relation for p_i:

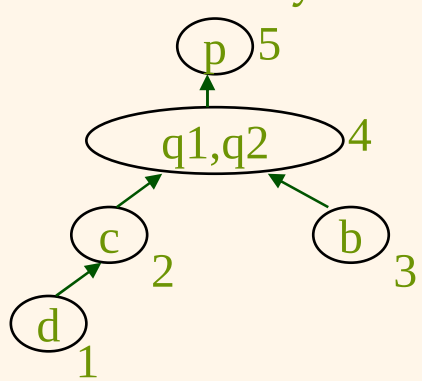


Initially: p_i , δp_i^{old} and δp_i^{new} = $\emptyset$

Seminaïve evaluation – Cont.

- Some observations
  - p_i — All known p_i facts.
  - δp_i^{old} — p_i facts (first) generated in previous iteration.
  - δp_i^{new} — p_i facts generated in this iteration.
  - p_i^{old} — p_i - δp_i^{old} (∴ generated before prev. iteration)

  NO ‘INFERENCE’ is ever repeated!
- A refinement of rule evaluation:
  - Go “node by node, bottom-up” in program graph.



Evaluation order = 1, 2, 3, 4, 5

or 3, 1, 2, 4, 5

Top-down evaluation

- Given:

  Call: Q(?)

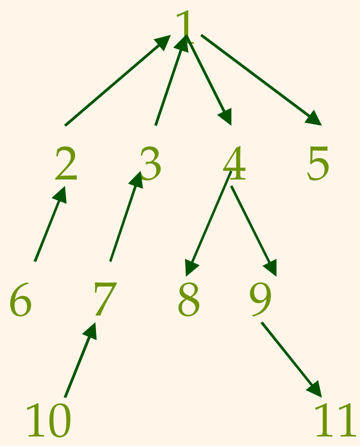
  Rule: Q IF $P_1 \wedge P_2 \ldots \wedge P_n$

  Generate subgoals:

  $P_1(?)\ P_2(?)\ \ldots P_n(?)$
- Advantage:
  - Computation is 'focused' in response to a query.
- Prolog is a language implemented in such a fashion.
  - Technique is called *resolution*

Example

up(2,1)	down(1,4)
up(3,1)	down(1,5)
up(6,2)	down(4,8)
up(7,3)	down(4,9)
up(10,7)	down(9,11)



r1: sg(X,Y) :- up(X,Z), down(Z,Y)

r2: sg(X,Y) :- up(X,Z_1), sg(Z_1,Z_2), down(Z_2,Y)

r3: sg(6,Y) ?

Prolog proceed as follows:

sg(6,y)_?

(r1)

up(6,Z)_?(Z=2); down(2,Y)_? fails;

up(6,Z)_? Fails on backtracking; (r1) fails.

(r2)

up(6,Z_1)_?(Z1=2);

sg(2,Z_2)_?

(r1)

up(2,Z’)_?(Z’=1); down(1,Y’)_?(Y’=Z_2=4);

sg(2,Z_2)_? succeeds with Z_2 =4;

down(4,Y)_?(Y=8);

sg(6,Y)_? succeeds with Y=8