Data Stream Processing (Part IV)

- Cormode, Muthukrishnan. "An improved data stream summary: The CountMin sketch and its applications", Jrnl. of Algorithms, 2005.
- Datar, Gionis, Indyk, Motwani. "Maintaining Stream Statistics over Sliding Windows", SODA'2002.
- *SURVEY-1:* S. Muthukrishnan. "Data Streams: Algorithms and Applications"
- *SURVEY-2:* Babcock et al. "Models and Issues in Data Stream Systems", ACM PODS'2002.

The Streaming Model

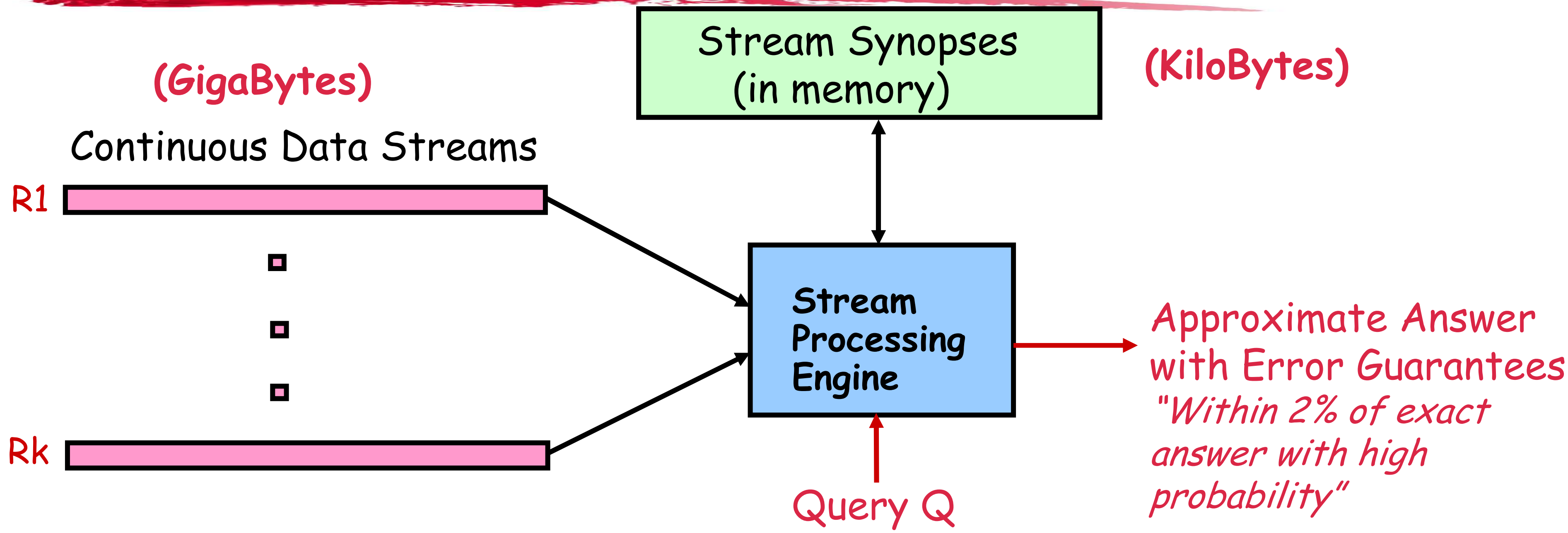
- Underlying signal: One-dimensional array A[1…N] with values A[i] all initially zero
  - Multi-dimensional arrays as well (e.g., row-major)
- Signal is implicitly represented via a stream of updates
  - j-th update is <k, c[j]> implying
    - A[k] := A[k] + c[j]   (c[j] can be >0, <0)

- Goal: Compute functions on A[] subject to
  - Small space
  - Fast processing of updates
  - Fast function computation
  - …

Streaming Model: Special Cases

- Time-Series Model
  - Only j-th update updates $A[j]$ (i.e., $A[j] := c[j]$)
- Cash-Register Model
  - $c[j]$ is always >= 0 (i.e., increment-only)
  - Typically, $c[j]=1$, so we see a multi-set of items in one pass
- Turnstile Model
  - Most general streaming model
  - $c[j]$ can be >0 or <0 (i.e., increment or decrement)
- *Problem difficulty varies depending on the model*
  - E.g., MIN/MAX in Time-Series vs. Turnstile!

Data-Stream Processing Model



- Approximate answers often suffice, e.g., trend analysis, anomaly detection
- Requirements for stream synopses
  - *Single Pass:* Each record is examined at most once, in (fixed) arrival order
  - *Small Space:* Log or polylog in data stream size
  - *Real-time:* Per-record processing time (to maintain synopses) must be low
  - *Delete-Proof:* Can handle record deletions as well as insertions
  - *Composable:* Built in a *distributed fashion* and combined later

Probabilistic Guarantees

- Example: Actual answer is within 5 ± 1 with prob ≥ 0.9
- Randomized algorithms: Answer returned is a specially-built random variable
- User-tunable (ε,δ)–**approximations**
  - Estimate is within a relative error of ε with probability >= $1-\delta$
- Use Tail Inequalities to give probabilistic bounds on returned answer
  - Markov Inequality
  - Chebyshev's Inequality
  - Chernoff Bound
  - Hoeffding Bound

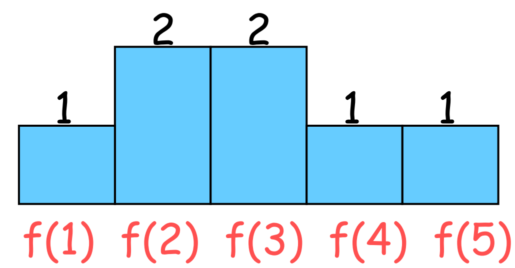
Overview

- Introduction & Motivation
- Data Streaming Models & Basic Mathematical Tools
- Summarization/Sketching Tools for Streams
  - Sampling
  - Linear-Projection (aka AMS) Sketches
    - *Applications:* Join/Multi-Join Queries, Wavelets
  - Hash (aka FM) Sketches
    - *Applications:* Distinct Values, Distinct sampling, Set Expressions

Linear-Projection (aka AMS) Sketch Synopses

- *Goal:* Build small-space summary for distribution vector f(i) (i=1,..., N) seen as a stream of i-values

Data stream: 3, 1, 2, 4, 2, 3, 5, . . .



- *Basic Construct: Randomized Linear Projection of f()* = project onto inner/dot product of f-vector

$$< f, \xi >= \sum f(i)\xi_i$$

where ξ = vector of random values from an appropriate distribution

  - Simple to compute over the stream: Add ξ_i whenever the i-th value is seen

Data stream: 3, 1, 2, 4, 2, 3, 5, . . . $\Rightarrow$ $\xi_1 + 2\xi_2 + 2\xi_3 + \xi_4 + \xi_5$

  - Generate ξ_i 's in small (logN) space using pseudo-random generators
  - *Tunable probabilistic guarantees* on approximation error
  - *Delete-Proof:* Just subtract ξ_i to delete an i-th value occurrence
  - *Composable:* Simply *add* independently-built projections

Hash (aka FM) Sketches for Distinct Value Estimation [FM85]

- Assume a hash function h(x) that maps incoming values x in [0,…, N-1] *uniformly* across [0,…, 2^L-1], where L = O(logN)

- Let lsb(y) denote the position of the least-significant 1 bit in the binary representation of y
  - A value x is mapped to lsb(h(x))

- Maintain *Hash Sketch* = BITMAP array of L bits, initialized to 0
  - For each incoming value x, set BITMAP[lsb(h(x))] = 1

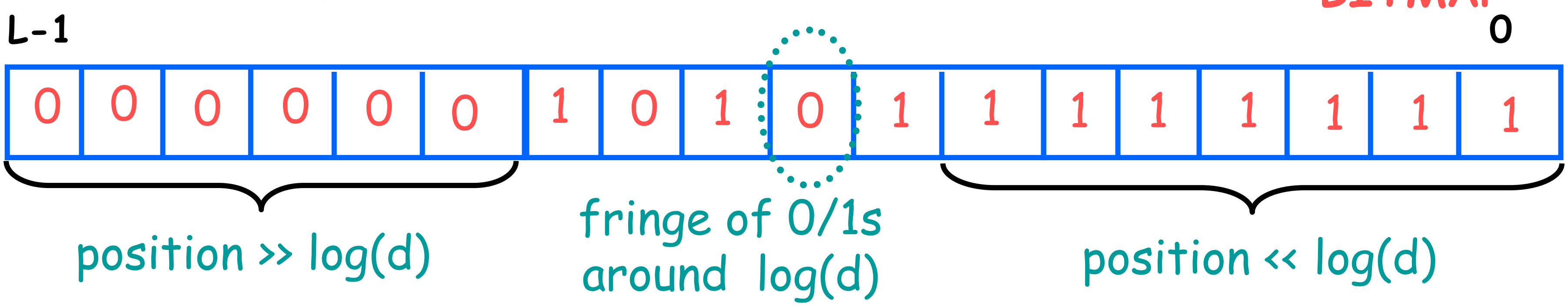
x = 5 → h(x) = 101100 → lsb(h(x)) = 2

BITMAP

5	4	3	2	1	0
0	0	0	1	0	0

Hash (aka FM) Sketches for Distinct Value Estimation [FM85]

- By uniformity through h(x): Prob[BITMAP[k]=1] = Prob[10^k] = $\frac{1}{2^{k+1}}$
  - Assuming d distinct values: expect d/2 to map to BITMAP[0] , d/4 to map to BITMAP[1], . . .



- Let R = position of rightmost zero in BITMAP
  - Use as indicator of log(d)
- Average several iid instances (different hash functions) to reduce estimator variance

Generalization: Distinct Values Queries

- SELECT COUNT(DISTINCT target-attr)
- FROM relation
- WHERE predicate

Template

- SELECT COUNT(DISTINCT o_custkey)
- FROM orders
- WHERE o_orderdate >= '2002-01-01'

TPC-H example

  - "How many distinct customers have placed orders this year?"
  - Predicate not necessarily only on the DISTINCT target attribute

- *Approximate answers with error guarantees over a stream of tuples?*

Distinct Sampling [Gib01]

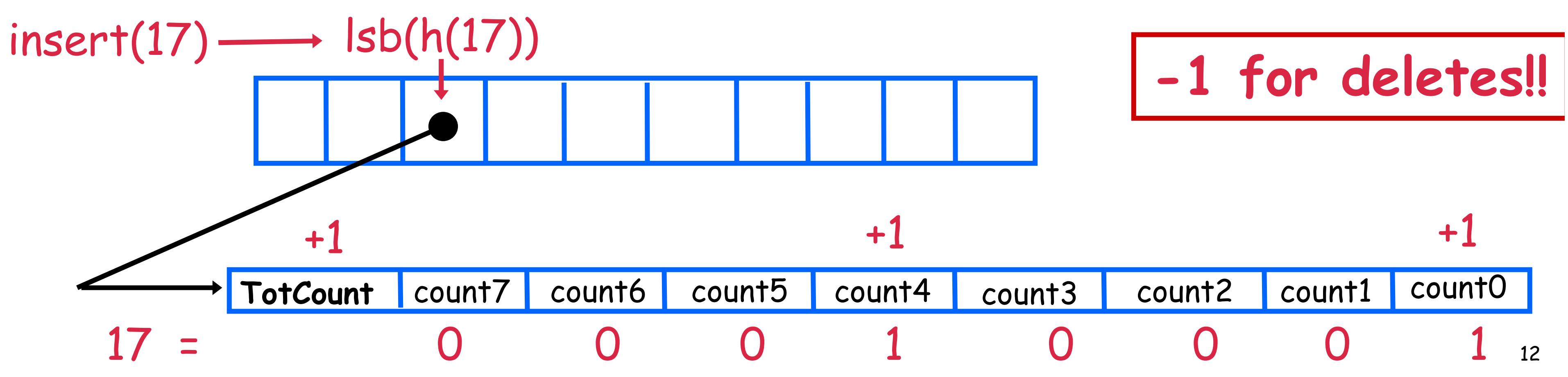
Key Ideas

- Use FM-like technique to collect a specially-tailored sample over the *distinct values in the stream*
  - ***Use hash function mapping to sample values from the data domain!!***
  - Uniform random sample of the distinct values
  - Very different from traditional random sample: each distinct value is chosen uniformly regardless of its frequency
  - DISTINCT query answers: simply scale up sample answer by sampling rate

- To handle additional predicates
  - *Reservoir sampling* of tuples for each distinct value in the sample
  - Use reservoir sample to evaluate predicates

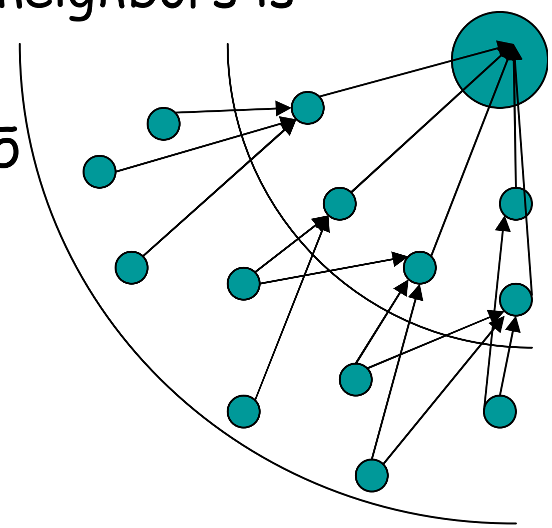
Processing Set Expressions over Update Streams [GGR03]

- Estimate cardinality of *general set expressions* over streams of updates
  - E.g., number of distinct (source,dest) pairs seen at both R1 and R2 but not R3? | (R1 $\cap$ R2) – R3 |
- *2-Level Hash-Sketch (2LHS) stream synopsis:* Generalizes FM sketch
  - *First level:* $\Theta(\log N)$ buckets with exponentially-decreasing probabilities (using lsb(h(x)), as in FM)
  - *Second level:* Count-signature array (logN+1 counters)
    - One "total count" for elements in first-level bucket
    - logN "bit-location counts" for 1-bits of incoming elements



Extensions

- Key property of FM-based sketch structures: ***Duplicate-insensitive!!***
  - Multiple insertions of the same value don't affect the sketch or the final estimate
  - Makes them ideal for use in broadcast-based environments
  - E.g., wireless sensor networks (broadcast to many neighbors is critical for robust data transfer)
  - Considine et al. ICDE'04; Manjhi et al. SIGMOD'05



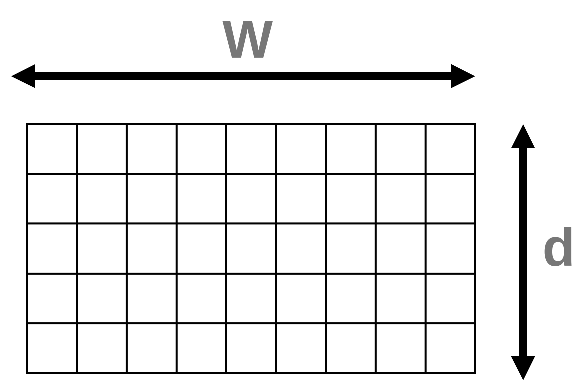
- Main deficiency of *traditional random sampling*: Does not work in a Turnstile Model (inserts+deletes)
  - "Adversarial" deletion stream can deplete the sample
- Exercise: Can you make use of the ideas discussed today to build a "delete-proof" method of maintaining a random sample over a stream??

New stuff for today…

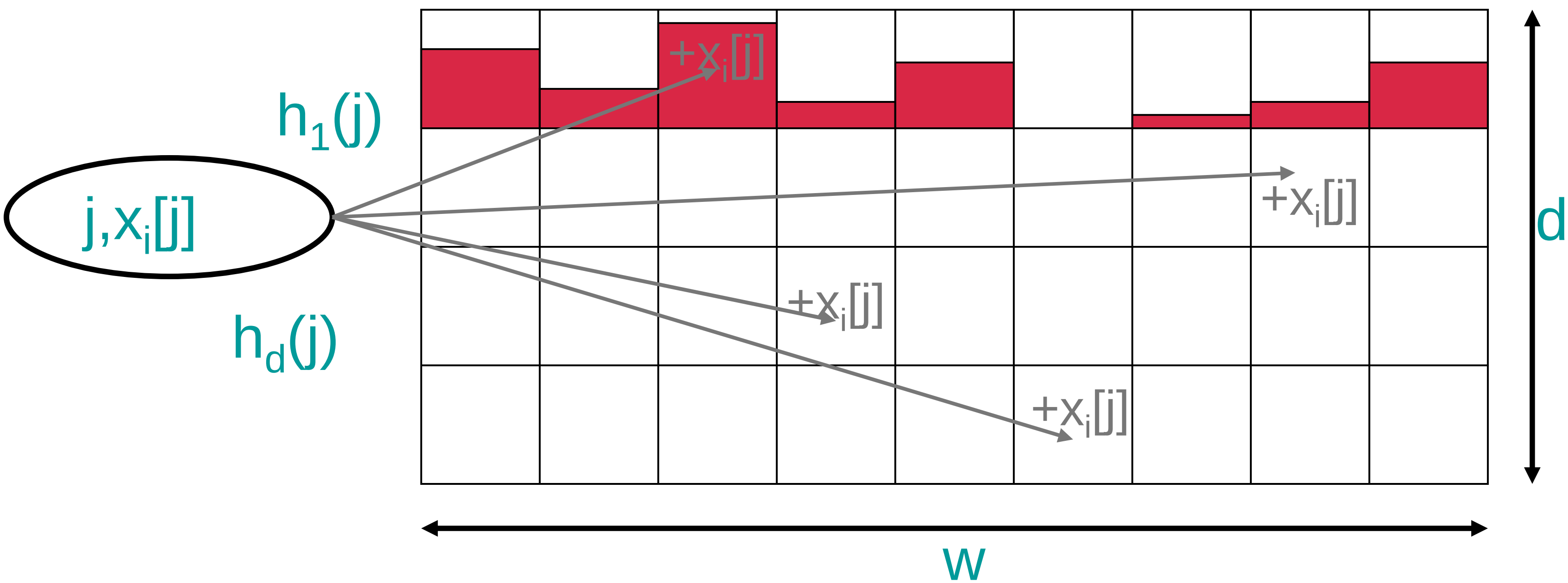
- A different sketch structure for multi-sets: *The CountMin (CM) sketch*
- The Sliding Window model and Exponential Histograms (EHs)
- Peek into distributed streaming

The CountMin (CM) Sketch

- Simple sketch idea, can be used for point queries, range queries, quantiles, join size estimation
- Model input at each node as a vector x_i of dimension N, where N is large
- Creates a small summary as an array of $w \times d$ in size
- Use d hash functions to map vector entries to $[1..w]$



CM Sketch Structure



- Each entry in vector *A* is mapped to one bucket per row
- Merge two sketches by entry-wise summation
- Estimate $A[j]$ by taking $\min_k \text{sketch}[k, h_k(j)]$

[Cormode, Muthukrishnan '05]

CM Sketch Summary

- CM sketch guarantees approximation error on point queries less than $\varepsilon||A||_1$ in size $O(1/\varepsilon \log 1/\delta)$
  - Probability of more error is less than $1-\delta$
  - Similar guarantees for range queries, quantiles, join size

- Hints
  - Counts are *biased!* Can you limit the expected amount of extra "mass" at each bucket? *(Use Markov)*
  - Use Chernoff to boost the confidence for the min{} estimate
- *Food for thought:* How do the CM sketch guarantees compare to AMS??

Sliding Window Streaming Model

- Model
  - At every time t, a data record arrives
  - The record "expires" at time t+N (N is the window length)
- When is it useful?
  - Make decisions based on "recently observed" data
  - Stock data
  - Sensor networks

Time in Data Stream Models

Tuples arrive $X_1, X_2, X_3, \ldots, X_t, \ldots$

- Function f(X,t,NOW)
  - Input at time t: $f(X_1,1,t), f(X_2,2,t). f(X_3,3,t), \ldots, f(X_t,t,t)$
  - Input at time t+1: $f(X_1,1,t+1), f(X_2,2,t+). f(X_3,3,t+1), \ldots, f(X_{t+1},t+1,t+1)$
- Full history: f == identity
- Partial history: Decay
  - Exponential decay: $f(X,t, NOW) = 2^{-(NOW-t)}*X$
    - Input at time t: $2^{-(t-1)}*X_1, 2^{-(t-2)}*X_2,, \ldots, \frac{1}{2} * X_{t-1},X_t$
    - Input at time t+1: $2^{-t}*X_1, 2^{-(t-1)}*X_2,, \ldots, 1/4 * X_{t-1}, \frac{1}{2} *X_{t,} X_{t+1}$
  - Sliding window (special type of decay):
    - $f(X,t,NOW) = X$ if $NOW-t < N$
    - $f(X,t,NOW) = 0$, otherwise
    - Input at time t: $X_1, X_2, X_3, \ldots, X_t$
    - Input at time t+1: $X_2, X_3, \ldots, X_{t,} X_{t+1,}$

Simple Example: Maintain Max

- Problem: Maintain the maximum value over the last N numbers.
- Consider all non-decreasing arrangements of N numbers (Domain size R):
  - There are ((N+R) choose N) distinct arrangements
  - Lower bound on memory required:
    log(N+R choose N) >= N*log(R/N)
  - So if R=poly(N), then lower bound says that we have to store the last N elements (Ω(N log N) memory)

Statistics Over Sliding Windows

- Bitstream: Count the number of ones [DGIM02]
  - Exact solution: $\Theta(N)$ bits
  - Algorithm BasicCounting:
    - $1 + \varepsilon$ approximation (relative error!)
    - Space: $O(1/\varepsilon\ (\log^2 N))$ bits
    - Time: $O(\log N)$ worst case, $O(1)$ amortized per record
  - Lower Bound:
    - Space: $\Omega(1/\varepsilon\ (\log^2 N))$ bits

Approach: Temporal Histograms

Example: … 01101010011111110110 0101 …

Equi-width histogram:

… 0110 1010 0111 1111 0110 0101 …

- Issues:
  - Error is in the last (leftmost) bucket.
  - Bucket counts (left to right): $C_m, C_{m-1}, \ldots, C_2, C_1$
  - Absolute error $\le C_m/2$.
  - Answer $\ge C_{m-1}+\ldots+C_2+C_1+1$.
  - Relative error $\le C_m/2(C_{m-1}+\ldots+C_2+C_1+1)$.
  - Maintain: $C_m/2(C_{m-1}+\ldots+C_2+C_1+1) \le \varepsilon$ $(=1/k)$.

Naïve: Equi-Width Histograms

- Goal: Maintain $C_m/2 \le \varepsilon\,(C_{m-1}+\ldots+C_2+C_1+1)$

Problem case:

… 0110 1010 0111 1111 0110 1111 0000 0000 0000 0000 …

- Note:
  - Every Bucket will be the last bucket sometime!
  - New records may be all zeros ➔
    For **every** bucket i, require $C_i/2 \le \varepsilon\,(C_{i-1}+\ldots+C_2+C_1+1)$

Exponential Histograms

- Data structure invariant:
  - Bucket sizes are non-decreasing powers of 2
  - For every bucket size other than that of the last bucket, there are at least k/2 and at most k/2+1 buckets of that size
  - Example: k=4: (8,4,4,4,2,2,2,1,1..)
- Invariant implies:
  - Assume $C_i=2^j$, then
    - $C_{i-1}+\ldots+C_2+C_1+1 >= k/2^*(\Sigma(1+2+4+..+2^{j-1})) >= k^*2^j /2 >= k/2^*C_i$
  - Setting $k = 1/\varepsilon$ implies the required error guarantee!

Space Complexity

- Number of buckets m:
  - m <= [# of buckets of size j]*[# of different bucket sizes]
    <= (k/2 +1) * ((log(2N/k)+1) = O(k* log(N))
- Each bucket requires O(log N) bits.
- Total memory:
  $O(k \log^2 N) = O(1/\varepsilon * \log^2 N)$ bits
- Invariant (with $k = 1/\varepsilon$) maintains error guarantee!

EH Maintenance Algorithm

Data structures:

- For each bucket: timestamp of most recent 1, size = #1's in bucket
- LAST: size of the last bucket
- TOTAL: Total size of the buckets

New element arrives at time t

- If last bucket expired, update LAST and TOTAL
- If (element == 1)
  Create new bucket with size 1; update TOTAL
- Merge buckets if there are more than k/2+2 buckets of the same size
- Update LAST if changed

Anytime estimate: TOTAL – (LAST/2)

Example Run

- If last bucket expired, update LAST and TOTAL
- If (element == 1)
  Create new bucket with size 1; update TOTAL
- Merge two oldest buckets if there are more than k/2+2 buckets of the same size
- Update LAST if changed

Example (k=2):

32,16,8,8,4,4,2,1,1

32,16,8,8,4,4,2,2,1

32,16,8,8,4,4,2,2,1,1

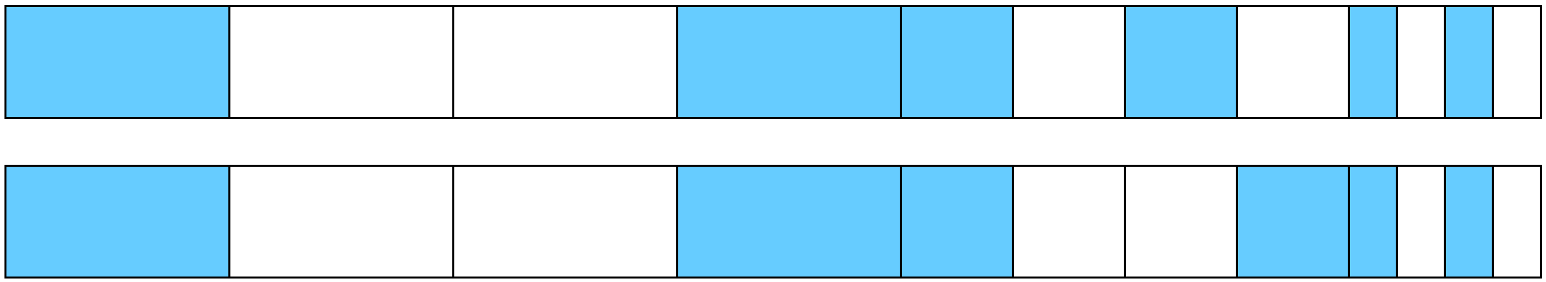
32,16,16,8,4,2,1

Lower Bound

- Argument: Count number of different arrangements that the algorithm needs to distinguish
  - $\log(N/B)$ blocks of sizes $B, 2B, 4B, \ldots, 2^iB$ from right to left.
  - Block i is subdivided into B blocks of size 2^i each.
  - For each block (independently) choose $k/4$ sub-blocks and fill them with 1.
- Within each block: (B choose $k/4$) ways to place the 1s
- (B choose $k/4$)$^{\log(N/B)}$ distinct arrangements

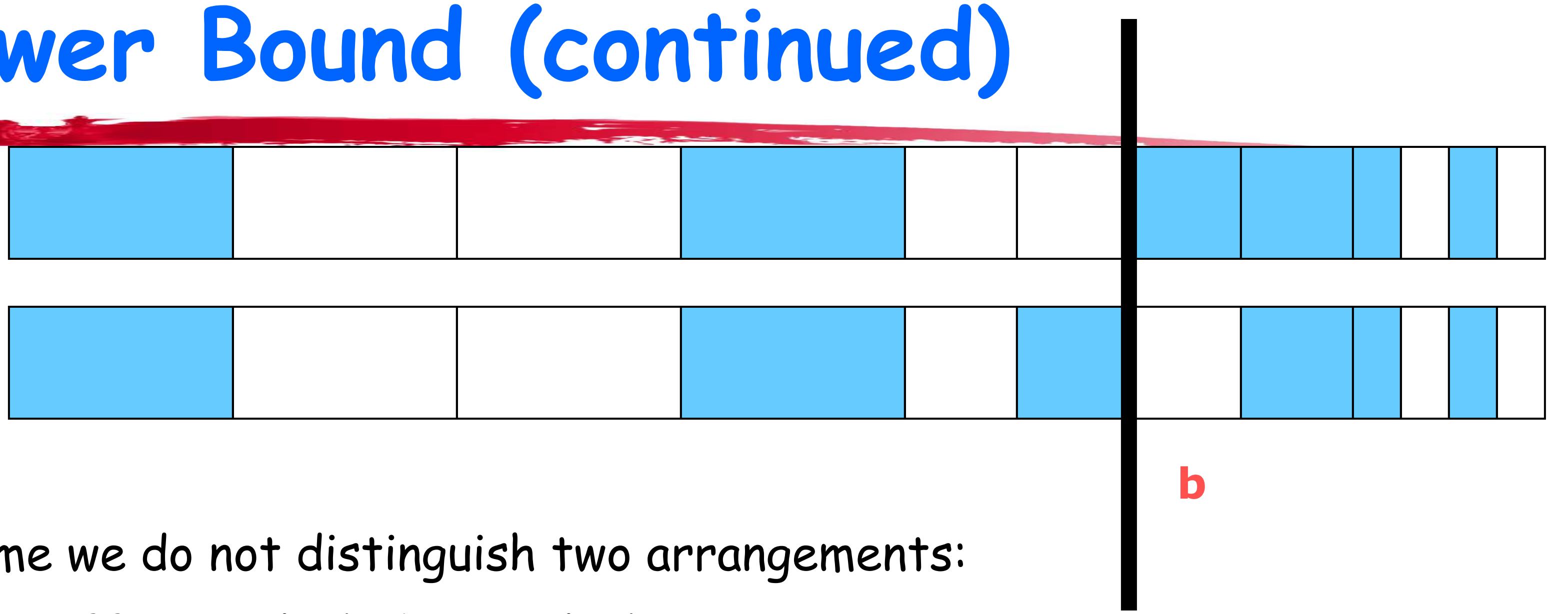
Lower Bound (continued)

- Example:



- Show: An algorithm has to distinguish between any such two arrangements

Lower Bound (continued)



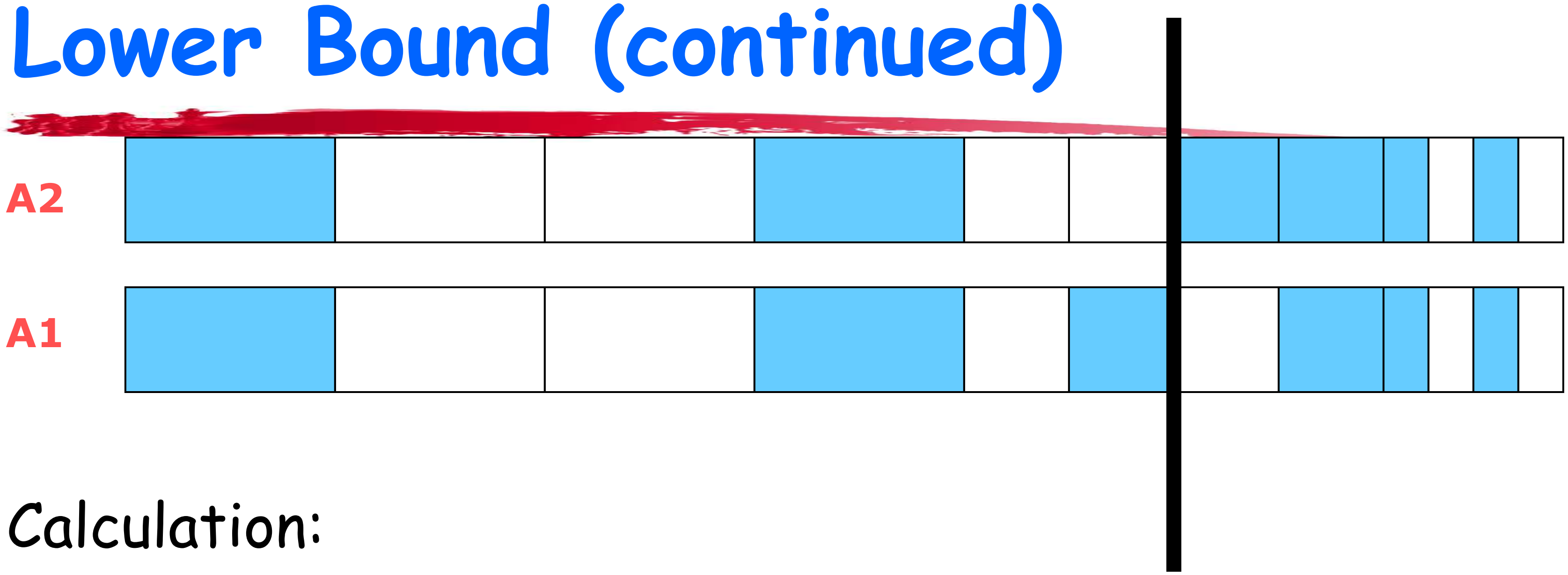
Assume we do not distinguish two arrangements:

- Differ at block d, sub-block b

Consider time when b expires

- We have c full sub-blocks in A1, and c+1 full sub-blocks in A2 [note: c+1<=k/4]
- A1: $c2^d$+sum1 to d-1 $k/4*(1+2+4+..+2^{d-1})$
  $= c2^d+k/2*(2^d-1)$
- A2: $(c+1)2^d+k/4*(2^d-1)$
- Absolute error: 2^{d-1}
- Relative error for A2:
  $2^{d-1}/[(c+1)2^d+k/4*(2^d-1)] >= 1/k = \varepsilon$

Lower Bound (continued)



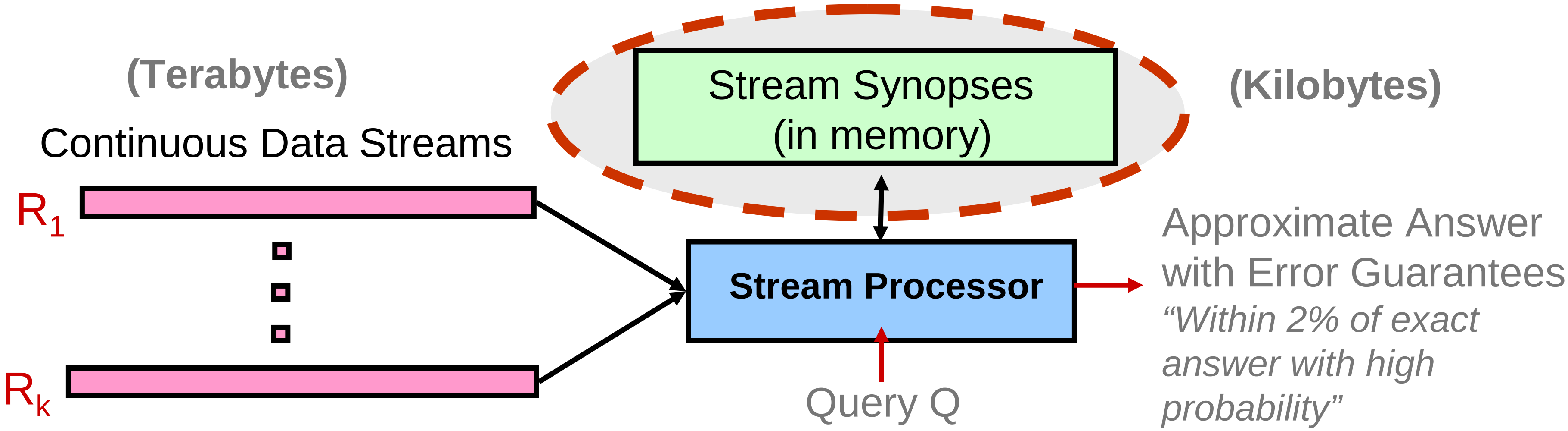
Calculation:

- A1: $c2^d$+sum1 to d-1 $k/4*(1+2+4+..+2^{d-1})$
  $= c2^d+k/2*(2^d-1)$
- A2: $(c+1)2^d+k/4*(2^d-1)$
- Absolute error: 2^{d-1}
- Relative error:
  $2^{d-1}/[(c+1)2^d+k/4*(2^d-1)]$ >=
  $2^{d-1}/[2*k/4*\ 2^d] = 1/k = \varepsilon$

The Power of EHs

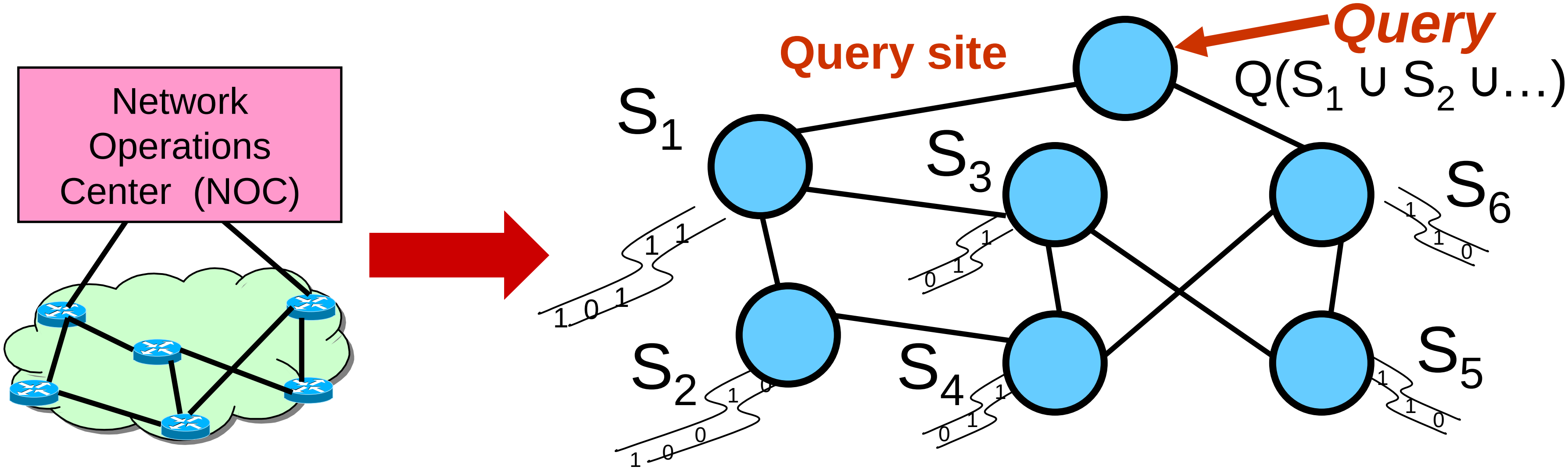
- Counter for N items = O(logN) space
- EH = ε–approximate counter *over sliding window* of N items that requires $O(1/\varepsilon * \log^2 N)$ space
  - O(1/ε logN) penalty for (approx) sliding-window counting
- Can plugin EH-counters to counter-based streaming methods → work in sliding-window model!!
  - Examples: histograms, CM-sketches, …
- *Complication:* counting is now ε–approximate
  - Account for that in analysis

Data-Stream Algorithmics Model



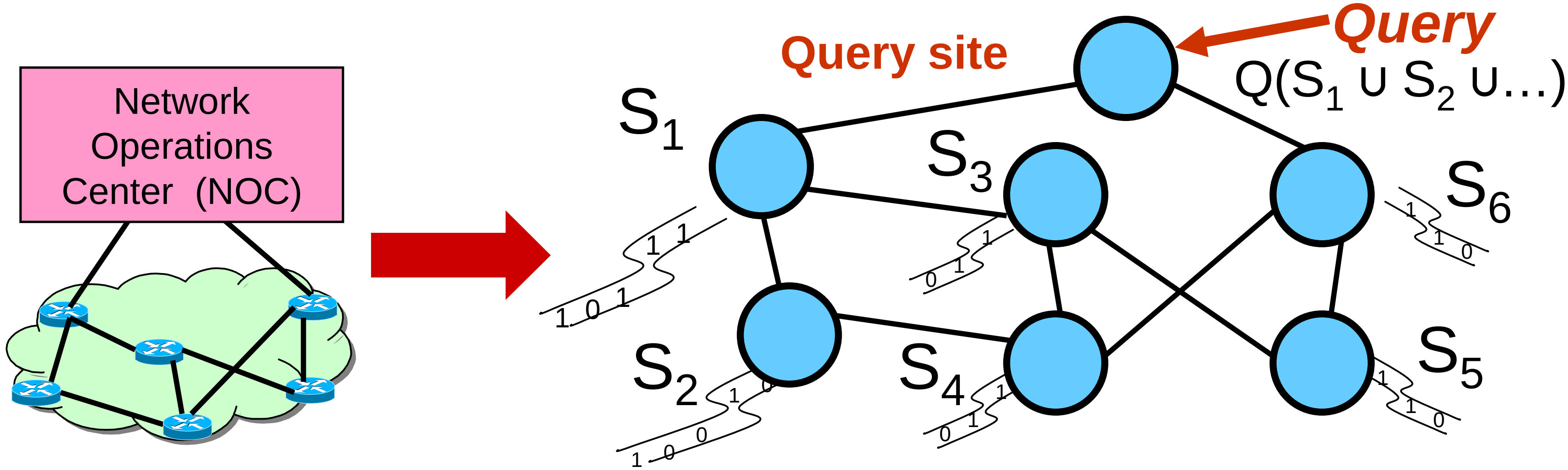
- *Approximate answers*– e.g. trend analysis, anomaly detection
- Requirements for stream synopses
  - *Single Pass:* Each record is examined at most once
  - *Small Space:* Log or polylog in data stream size
  - *Small-time:* Low per-record processing time (maintain synopses)
  - Also: *delete-proof, composable, …*

Distributed Streams Model



- Large-scale querying/monitoring: *Inherently distributed!*
  - Streams physically distributed across remote sites
    E.g., stream of UDP packets through subset of edge routers
- *Challenge is "holistic" querying/monitoring*
  - Queries over the *union of distributed streams* $Q(S_1 \cup S_2 \cup \ldots)$
  - Streaming data is spread throughout the network

Distributed Streams Model



- Need timely, accurate, and efficient query answers
- Additional complexity over centralized data streaming!
- Need space/time- *and communication-efficient* solutions
  - Minimize network overhead
  - Maximize network lifetime (e.g., sensor battery life)
  - Cannot afford to “centralize” all streaming data

Conclusions

- Querying and finding patterns in massive streams is a real problem with many real-world applications
- Fundamentally rethink data-management issues under stringent constraints
  - Single-pass algorithms with limited memory resources
- A lot of progress in the last few years
  - Algorithms, system models & architectures
    - GigaScope (AT&T), Aurora (Brandeis/Brown/MIT), Niagara (Wisconsin), STREAM (Stanford), Telegraph (Berkeley)
- Commercial acceptance still lagging, but will most probably grow in coming years
  - Specialized systems (e.g., fraud detection, network monitoring), but still far from "DSMSs"